DeepDoom: Navigating Complex Environments Using Distilled Hierarchical Deep Q-Networks

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[](https://github.com/Atlas-Soft/DeepDoom)

[Append System specs?]

*Abstract*— [Write After Completing Paper]

*Index Terms*—Deep Reinforcement Learning (DRL), Deep Q-Networks (DQN), Hierarchical Deep Q-Networks (h-DQN), Policy Distillation

# INTRODUCTION

D

ESPITE improvements to game-playing AI, agents rely on looking “under the hood” of their respective games in order to gain access to the various internal variables, thus holding an unfair advantage over their human counterparts. However, ally and enemy AIs are typically designed to keep the human player engaged with a fair challenge. When set on easier difficulty settings, AIs are typically considered “pushovers,” while in harder difficulty settings, AIs play nearly “perfect”. In order to design a more balanced and enjoyable experience, it may be more suitable for agents to behave similar to a human player using the same available environmental information.

Google DeepMind’s paper, *Playing Atari With Deep Reinforcement Learning* [Playing Atari…], shows the feasibility of game playing using only visual input. This was done by combining Deep Convolutional Neural Networks (CNNs) with Q-Learning, forming Deep Q-Networks (DQNs). Agents trained via DQNs were able to learn and play two-dimensional (2D) Atari 2600 games such as Pong, Breakout, and Space Invaders. In other words, DQNs guided the agent towards the best policy using positive or negative responses from their respective environments. Since then, there has been an ample amount of research into applying these same reinforcement learning techniques to train agents within three-dimensional (3D) environments such as Minecraft [The Malmo…] and Doom.



*Figure 1 - A sample Doom environment*

The Doom series [Doom (series)] began in 1993 with the release of the first game, DOOM, and it has since sparked countless adaptations of 3D first-person shooter (FPS) games. These environments heightened the level design complexity with the incorporation of depth, providing another factor when designing agents. The relevancy of Doom with Artificial Intelligence can be contributed to the AI research platform ViZDoom [ViZDoom: A…] alongside the scenario customization software of Doom Builder and Slade (of which we used Doom Builder). ViZDoom allows programmers to test reinforcement learning techniques within the environment of Doom. Its Visual Doom AI Competitions [ViZDoom], as well as previous research [Playing FPS…], proved the feasibility of teaching agents to sufficiently play Doom using DQNs. However, they mainly focused on the combat aspect of Doom, with only a minor focus on the navigation aspect (via item and health pickups).

For our capstone project, we propose to create an agent capable of solving complex navigational problems using a hierarchical implementation [A Deep…] of the Deep Q-Networks, expanding on ViZDoom’s experimentations except with a heavier focus on navigation. We will first utilize DQN models to separately train simple tasks (such as turning and finding exits) and then integrate these as sub-models in a Hierarchical Deep Q-Network (h-DQN) model. In addition, we will investigate distilling, or compressing, trained h-DQN models into the simple DQN architecture for a more resource-efficient execution. Increasingly-complex behaviors can then be achieved by incrementally-aggregating distilled h-DQN models with other skills into other Hierarchical-DQN models, thus reflecting our goal of developing a more proficient Doom-playing Artificial Intelligence.

# RELATED WORK

The inspiration for the use of Deep Reinforcement Learning (DRL) techniques in the context of video games looks to Mnih et al. and the Atari 2600 games, where DQNs (a composition of CNNs and Q-Learning) were used to learn control policies from several games [Playing Atari…]. This demonstrates the applicability of DRL techniques towards learning the various skills needed to play games in a 2D virtual environment. From there, the jump from a 2D to a 3D virtual environment was not inconsiderable, as both environments represented high-dimensional problem spaces.

A recent attempt at visual navigation includes a focus on color and texture to navigate maze-like environments using an otherwise similar approach to Mnih et al. [Learning to...]. This demonstrates that it is feasible to train a CNN capable of navigating complex environments using only visually-apparent data. However, this leaves out some of the inherent complexity of a video game such as Doom, where the player is required to use multiple task-specific skills in differing scenarios to reach the end of a level (like operating in-game objects). In other words, while navigation is certainly possible, the difficulty of training an DQN-agent to play FPS games increases as the amount of skills required to complete levels increase.

Fortunately, ViZDoom was created as an experimental platform for the development of visual-input-based agents [ViZDoom: A...]. ViZDoom uses the ZDoom engine to provide an environment to train agents within the context of a first-person shooter game—in this case, Doom and its derivative games. This allows for an extreme amount of customizability in regards to training scenarios, because users can create their own levels to train agents on. To demonstrate the functionality of their platform, they created a simple bot using a DQN model similar to Mnih et al.

Kempka et al. took this a step further by using the ViZDoom platform to construct a bot capable of Deathmatch scenarios [Playing FPS...]. Kempka et al. decomposed the Deathmatch task into individual skills and trained separate Deep Q-Networks for shooting and navigation tasks after realizing that a single DQN would be incapable of adequately completing the scenario. Combining these individually learned skills proved that an agent could be trained to perform in Deathmatch scenarios by simplifying the global task in simpler local skills.

Hierarchical Deep Q-Networks (h-DQN) were shown by Zhou, Kampen, and Chu to demonstrate task decomposition, while also helping accelerate learning and more efficiently transfer knowledge between tasks within the context of flight navigation [Autonomous Navigation...]. Their h-DQN was able to handle the global task of navigation while a sub-model performed obstacle avoidance. With task decomposition in mind, we looked toward a more robust and adaptive technique, such as h-DQN, as opposed to alterations to the Q-Learning functions.

Additionally, Tessler et al showed that h-DQN models can be trained to selectively use a collection of subnetworks to accomplish high-level tasks in a way that outstrips DQN and Double DQN (DDQN) models in learning performance and convergence rate within the context of Minecraft [A Deep...]. Not only does this demonstrate the applicability of their model to complex 3D environments, but it is also remarkably similar to the kind of decision making we believe necessary to play Doom, because subnetworks are managed such that they are only used as needed.

Our intent is to incorporate the structural capability of h-DQNs with Distillation, which traditionally allows for the transfer of knowledge from network ensembles or large, regularized network models to smaller networks, as shown by Hinton, Vinyals, and Dean [Distilling the...]. This allows for the use of inefficient, albeit more accurate models during training, while switching over to more compact models for product deployment. Despite the information loss due to compression, they show that a distilled network performs comparably to its larger counterparts.

Rusu et al. shows that policies from single task-oriented DQNs can be successfully distilled or compressed in the context of Atari 2600 games [Policy Distillation]. In this case, task distillation allows a distilled DQN to play multiple games better than a standard DQN trained to perform multiple tasks. By decomposing gameplay into individual skills in Doom, we aim to apply policy distillation in a similar manner, but with a focus on the modularity afforded to us by the hierarchical model. This should allow us to effectively distill an h-DQN for use as a sub-model in another h-DQN – thereby treating the cumulative knowledge of an h-DQN as an individual skill.

# Background ??

Below we give brief description of our initial work, DQNs, h-DQNs and h-DQN distillation.

## Initial Work

We initially explored using a supervised learning and monte-carlo tree search as its been shown to be effective in video game environments(cite). Our goal was for the agent to be able to complete Doom levels designed for human players. The idea here was to visually train our agent based solely on human gameplay of Doom. This method was used in an attempt to learn the policy of original Doom II levels. In hindsight, this methodology of agent training was extremely rigid. The agent was limited to simply imitating human actions, having no feedback on whether some action is appropriate or not. Initial agent training on the first level provided mixed results; the agent successfully traversed basic navigational pieces of the level, such as rooms and corridors, but faced slight difficulties recognizing doors. Specifically, the agent would appear to confuse wall textures as doors as the agent would consistently run into some sections of the corridor walls. The complexity of even the first level of Doom proved much for our initial supervised model which led us to seek more effective methods of building an agent, namely Deep Q-Networks.

## Q-Learning

Q-Learning is a type of reinforcement learning that defines a function, **Q(s,a)**, which determines the maximum possible reward, **R**, at a time, **t**, of a future state by performing an action, **a**, on a state, **s**. Or:

**Q(st,at) = max Rt**

Within the context of Doom, an action, **a**, would be a command input, such as moving forwards, backwards, side-to-side, turning, and activating an object. A state, **s**, would be a frame of visual data. The reward, **R**, is a value we define to be awarded to the bot upon meeting certain conditions, depending on the scenario it is operating in. Lastly, the time, **t**, simply says that a reward corresponds to a particular action and state.

These Q-values can then be used to determine the policy, or best action, ***π***, at a given state, **s**, by,

***π*(s) = argmaxa Q(s)**,

where **argmaxa** is the index of the action, **a**, with the highest Q-value, **Q(s,a)**.

To approximate the value of **Q(s,a)** the Bellman Equation is used to update the Q-function,

**Q(s,a) = Q(s,a) + *α*(R + *g* \* (maxa' Q(s')) - Q(s,a))**,

where **a'** and **s'** are the next action and state used to determine a Q-value, ***α*** is the Q-learning rate between 0.0 and 1.0, ***g***is the discount factor between 0.0 and 1.0, and **maxa' Q(s')** is the index of the next action, **a'**, with the highest Q-value. To further clarify some of these parameters, if ***α* = 0**, then the Q-function will remain the same. Furthermore, ***g*** determines how heavily the next highest valued action is considered, and at ***g* = 0**,thenext highest valued action will not be considered at all, where at ***g* = 1** the next highest valued action will be heavily considered. While the approximation of **Q(s,a**) may be completely wrong in the early stages of training, it will eventually converge to represent true Q-values.

## Deep Q-Networks

[Explain our Q-Network with CNN? Input>Convx32>Convx64>FC>Actions; activated using ReLU]

[Describe reasons for choices:

* Replay Memory
* Filter sizes determined by VizDoom: A...
* Number of filters determined by system specs; most w/o impacting training time
* FC Layers 4032 nodes = # of nodes at the end of the last conv layer when flattened
  + Flatten: takes 2d set of nodes, and makes them 1d
* Resolution = 120x160
* ReLU instead of Tanh or Sig
* Mean-Squared Err Loss Function
* Optimizer: RMSprop LR = .0001
* Dropout = .5
* Depth radius and contrast

A DQN uses multiple hidden layers and the Q-learning function to determine a policy, or set of actions, based on the Q-learning function. Our DQNs consist of an Input, two Convolutional Layers, a Fully Connected Layer, and an Output.

This task necessitated Convolutional Layers, which we used to abstract features from images to approximate the Q-function, because of our focus on visual data. Additionally, we used Replay Memory to reduce the instability commonly associated with using CNNs to compute Q-values. This involves using a random batch from a list of transitions to train the DQN, while ensuring it does not get stuck at a local minimum. Transitions take the form of,

**(s,a,r,s')**,

where **s** is the current state, **a** is the current action, **r** is the current reward, and **s'** is the previous state. Furthermore, we based our CNN filter sizes on the research done by Kempka et al., and our number of filters, 32 and 64 for each Convolutional Layer respectively, on what could be trained in a reasonable time given our system specs [see appendix].

After flattening the nodes in our last Convolutional Layer, which involves taking what is in essence a 2D matrix of nodes and converting them into a 1D array, there are 4032 nodes, which also determines the size of our Fully Connected Layers. Within our Fully Connected Layers we also applied a Dropout value of .5, which randomly removes a node from the network with a probability of .5 to help prevent over training.

Additionally, it is important to note that our layers are activated using Rectified Linear Units, which bring non-linearity into the network by thresholding our activations at zero, and we used the Mean-Squared Error loss function and a RMSProp Learning Rate value of .0001 to optimize our parameter values.

[add below to preprocessing section]

The input of our DQNs consists of a number of frames – 120x160 in resolution – a list of available actions, such as directional movement keys, a depth radius between 0.0 and 1.0 – which determines how far a bot can 'see' – a depth contrast between 0.0 and 1.0 – which determines how well a bot detects textures – and a boolean value - which determines whether or not the DQN will be distilled, and is set to False by default.

## Hierarchical Deep Q-Networks

* Diagram
* **Q(s,b) = Q(s,b) + *α*(R + *g* maxa' Q(s') - Q(s,b))**
* **B is submodel**

## Distillation

* Teacher and Student Network
* Transfer knowledge for Heirarchical model to simple DQN model.
* Data set for policy distillation is generated by taking the softmax normalization of the selected submodel.
* B =
* Qdistilledtargets(S) = softmax(Qb(S))

# TRAINING AND EXPERIMENTS

## Rigid Turning

The first scenario we began work on is very simple; the player is placed into a hallway with 90° turns, making the corridor S-shaped. The player starts at one end of the corridor and must navigate through the hall to the opposite end. To facilitate in the training process, we defined multiple lines throughout the level which increase a reward when the player crosses over them. These rewards can only be received once per each line definition, in order to prevent reinforcing a behavior of crossing back and forth across a line to maximize the reward. This reward function essentially pushes the agent through the corridor as it learns why it is rewarded. Additionally, there is a function that decreases the agent's reward when it touches a wall. This "reward" can be received indefinitely, and as such holds a relatively low value. As a result, the agent's traversal of the hallway more closely resembles that of a human player, and tends to give the agent more information to be used by the hierarchical model. In an effort to reduce the likelihood of over training the model, the textures of the wall, floor, and ceiling textures are randomly chosen from a pre-selected pool of several of the game’s included textures.

## Exit Finding

Creating a scenario to facilitate in how to learn to find exits from a room proved more complicated than originally anticipated. The map we are using consists of a large square room, which the player starts in, with a narrow but long corridor leading out from one edge, which the player must enter to end the scenario. Initially, this narrow corridor was not nearly as long, but this resulted in the agent having difficulty when starting training in distinguishing the depth of the corridor from the depth of the wall next to it. We did not feel that keeping the player’s starting position at a fixed point would very accurately reflect the skill required for finding an exit out of a room. In such a situation, the network would likely be over trained, suitable only for turning *x* degrees and moving *y* units. Instead, we devised a script which would place the player in a random position in the room, facing a random direction.

Rewarding the agent also required slightly more ingenuity to be done accurately. The method of rewarding the agent for crossing over crossing over a line definition was no longer feasible when the starting position of the agent was made random. If the agent were placed in front of the exit, its reward would not be maximized by going directly to the reward, but rather by moving in the room to find the line definitions and then going to the exit. Originally, this was resolved by a continuously running a script that checked if the agent was moving towards the exit based on the distance between the point at the center of the exit and the *x* and *y* coordinates of the player. While this reward did guide the agent towards the exit, it inadvertently rewarded the agent for moving as slowly as it was capable, as this would result in more frames that the reward function would be satisfied, therefore inflating the reward received.

The reward function was greatly overhauled upon discovering the above unwanted behavior. In the exit\_finding\_v2 file, the reward function is applied via discrete line definitions, progressively arching outwards from the exit. These line definitions have attached scripts which borrow from the ideas present in the dynamically calculated rewards. When activated by walking over an associated line definition, the script first checks if the player is looking at the exit by comparing the orientation of the player to the angle of the exit relative to the player. If the player is facing the exit and is moving forward while crossing a line definition, then the agent is rewarded, with the reward increasing as the distance between the agent and the exit decreases. Because having curved line definitions is not possible in the Doom engine, the arches were comprised of multiple line definitions approximating a curve. The staggered nature of the reward value aided in overcoming the problems with having multiple line definitions, and a flag was created to stop rewards for each “tier” after they had been rewarded. When the player has reached the exit, they are given a reward, and the scenario ends.

## Door Opening??

Creating an environment where the agent could learn how to open doors was straightforward at first glance. However, to prevent overtraining from occurring, a degree of randomness is required. The map for this scenario has a very simple layout, being a single corridor with nine doors placed equidistant from each other along the length of the hallway. The textures of the doors are assigned randomly at the time the level loads from a pool of pre-selected door textures. Likewise, the walls, floor, and ceiling are textured in the same manner.

When determining how to reward the agent, the most obvious method resulted in unwanted behavior. Our first revision of the reward function rewarded the agent for activating/opening a door, but this resulted in the agent repeatedly pressing the “use” action on the door, constantly opening and closing it, instead of progressing towards the end of the scenario. To eliminate this behavior, the reward function was made closer to the reward function of the rigid turning scenario. To promote advancing through the scenario, the agent was rewarded for crossing line definitions placed in doorways, and for crossing lines placed between two doors to promote moving towards the next door. Each reward was given only once.

## Switch Pressing - Deceased??

Upon deciding to implement sub models for individual skills, we decided that recognition and use of switches would require its own model, but this was ultimately not necessary. Due to the reward function of the composite scenario, the agent learned to adapt the door opening skill to press switches. This realization rendered the switch pressing model obsolete, and thus it was not trained. The proposed map for this model is similar to the exit finding scenario. The player is placed in a small square room, with a switch placed in the middle of the south wall. Upon the player entering the map, the player’s position and orientation is randomly assigned, and the textures of the walls, ceiling, floor, and the switch are selected randomly from a pre-determined pool of textures.

To reward the agent, line definitions are placed arching around the switch, spaced 32 units apart. The script for these line definitions is identical to those of the exit finding scenario, only providing their reward when the player is both facing and moving towards the switch. Like in exit finding, the reward is increased as distance between the player and switch decreased, and a specific reward value is only given once.

## Comprised Skills??

# RESULTS

## Custom Scenarios??

## Hierarchical-DQN vs. DQN??

## Distillation Benefits??

# CONCLUSION

## Project Thoughts??

## Reward System??

## Future Work??

# ACKNOWLEDGEMENTS

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